Vortex sub-lattice melting in a two-component superconductor

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We consider the vortex matter in a three-dimensional two-component superconductor with individually conserved condensates with different bare phase stiffnesses in a finite magnetic field, such as the projected superconducting state of liquid metallic hydrogen. The ground state is an Abrikosov lattice of *composite*, i.e. co-centered, vortices in both order parameters. We investigate quantitatively two novel phase transitions when temperature is increased at fixed magnetic field. *i)* A "vortex sub-lattice melting" phase transition where vortices in the field with lowest phase stiffness ("light vortices") loose co-centricity with the vortices with large phase stiffness ("heavy vortices"), thus entering a liquid state. Remarkably, the structure factor of the light vortex sub-lattice vanishes *continuously*. This novel transition, which has no counterpart in one-component superconductors, is shown to be in the 3Dxy universality class. Across this transition, the lattice of heavy vortices *remains intact*. *ii)* A first order melting transition of the lattice of heavy vortices, with the novel feature that these are *interacting with a background liquid of light vortices*. These findings are borne out in large-scale Monte Carlo simulations.

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Theories with multi-component bosonic scalar matter fields minimally coupled to a gauge field are of interest in a variety of condensed matter systems and beyond. This includes such disparate systems as superconducting low temperature phases of light atoms[1,2,3,4,5,6] under extreme enough pressures to produce liquid metallic states, easy-plane quantum antiferromagnets[7], as well as other multiple-component superconductors[2,3]. It also has applications in particle physics[8]. The projected liquid metallic states of hydrogen (LMH)[1] may soon be realized in high pressure experiments[9,10]. Moreover, LMH is abundant in the interiors of the giant planets Jupiter and Saturn, where it is the origin of their magnetospheres[11]. At low temperatures, compressed liquid hydrogen is particularly interesting since it features prominent quantum fluctuations which lead to the possibility of a new state of matter, a near ground state liquid metal[1]. Its superconducting counterpart involves Cooper pairs of electrons and protons[1], whence symmetry precludes Josephson coupling between different condensate species. Resolving what happens to such a system in a magnetic field is now a matter of some urgency, due to new and detailed first principles calculations predicting LMH under extreme pressures of order $400 GPa$[10]. This is not far from experimentally achieved pressures of $320 GPa$[9], where hints of a maximum in the melting temperature versus pressure are evident. Magnetic-field experiments may very likely be exclusive probes to provide confirmation of LMH. A first study of the phase diagram of the projected LMH in magnetic fields has been presented, unveiling a phase diagram with rich structure[5]. This study raises complex issues of interest also in the broader domain of physics concerning the order and universality classes of possible phase transitions separating phases of partially broken symmetries in quantum fluids. Here, we report on a study of this question based on a confluence of exact topological arguments and large-scale Monte-Carlo(MC) simulations.

For general number of components N (relevant for mixtures of light atoms), the Ginzburg-Landau model is defined by the Lagrangian

$$\mathcal{L} = \sum_{\alpha=1}^{N} \frac{|\mathbf{D}\Psi_0^{(\alpha)}(\mathbf{r})|^2}{2M^{(\alpha)}} + V(\{\Psi_0^{(\alpha)}(\mathbf{r})\}) + \frac{1}{2}(\nabla \times \mathbf{A}(\mathbf{r}))^2. \quad (1)$$

Here, $\{\Psi_0^{(\alpha)}(\mathbf{r}) \mid \alpha = 1 \dots N\}$ are complex scalar fields, $M^{(\alpha)}$ is the mass of the condensate species α, and $\mathbf{D} = \nabla - \mathrm{i}e\mathbf{A}(\mathbf{r})$. In LMH *each individual condensate is conserved*, consequently $V(\{\Psi_0^{(\alpha)}(\mathbf{r})\})$ is only a function of $|\Psi_0^{(\alpha)}(\mathbf{r})|^2$. The model is studied in the phase-only approximation $\Psi_0^{(\alpha)}(\mathbf{r}) = |\Psi_0^{(\alpha)}| \exp[\mathrm{i}\theta^{(\alpha)}(\mathbf{r})]$ where $|\Psi_0^{(\alpha)}| = \mathrm{const.}$

For the discussions in this paper, another form of the action is useful. Introducing $|\psi^{(\alpha)}|^2 = |\Psi_0^{(\alpha)}|^2/M^{(\alpha)}$ and $\Psi^2 \equiv \sum_{\alpha=1}^{N} |\psi^{(\alpha)}|^2$, Eq. (1) may be rewritten[6] in terms of *one* charged and $N-1$ neutral modes

$$\begin{aligned}\mathcal{L} \;&=\; \frac{1}{2\Psi^2}\left(\sum_{\alpha=1}^{N} |\psi^{(\alpha)}|^2 \nabla\theta^{(\alpha)} - e\Psi^2\mathbf{A}\right)^2 + \frac{1}{2}(\nabla \times \mathbf{A})^2 \\ &+\; \frac{1}{4\Psi^2}\sum_{\alpha,\beta=1}^{N} |\psi^{(\alpha)}|^2\, |\psi^{(\beta)}|^2 \,\left(\nabla(\theta^{(\alpha)} - \theta^{(\beta)})\right)^2. \quad (2)\end{aligned}$$

While Eq. (1) is convenient for MC simulations, Eq. (2) has advantages for analytical considerations, since the neutral and the charged modes are explicitly identified. Moreover, Eq. (2) is convenient for identifying various states of partially broken symmetry, emerging when an N-component system is subjected to an external magnetic field[5,6].

From now on we focus on the case $N = 2$. In the case of LMH, $\Psi_0^{(1)}$ and $\Psi_0^{(2)}$ will denote protonic and electronic superconducting condensates, respectively, and hence $|\psi^{(1)}|^2 \ll |\psi^{(2)}|^2$. In zero external magnetic field this system features a low-temperature phase-transition in the 3Dxy universality class at T_{c1} where superfluidity is lost, followed at higher temperatures by an inverted 3Dxy transition at T_{c2} where superconductivity and the Higgs mass of $\mathbf{A}$ (Meissner effect) is lost[4]. Here, we will consider the system in finite magnetic field at temperatures below T_{c2}[5].

We define a type-α vortex as a topological defect in $\Psi_0^{(\alpha)}$ associated with a non-trivial phase winding $\Delta\theta^{(\alpha)} = \pm 2\pi$, whereas a *composite vortex* is a topological defect where type-1 and type-2 vortices coincide in space. At low temperatures the formation of an Abrikosov lattice of *non-composite vortices* is forbidden because these vortices have a logarithmically divergent energy, whereas *composite* vortices have finite energy[3,4]. In a type-II 2-component superconductor, therefore, an Abrikosov lattice of composite type-1 and type-2 vortices is formed, illustrated in panel A of Fig. 1. At elevated temperatures, the 2-component system in a magnetic field will exhibit thermal excitations in the form of fractional-flux vortex loops similar to the case of zero magnetic field $\mathbf{B} = \nabla \times \mathbf{A} = 0$[3,4]. Since the field-induced vortices are logarithmically bound states of constituent (elementary) vortices, the thermal fluctuations will induce a *local splitting* of composite vortices in the form of two half-loops connected to a straight line[5], as shown in panel B of Fig. 1.

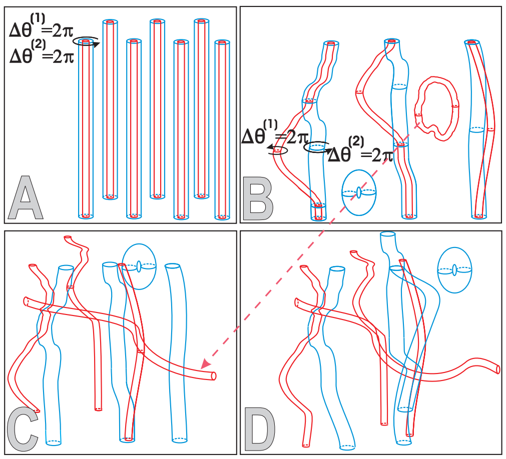

FIG. 1: **Panel A:** A type-II, $N = 2$ superconductor at zero temperature in a magnetic field forms a lattice of composite Abrikosov vortices, i.e. co-centered type-1 (red) and type-2 (blue) vortices. **Panel B:** Low-temperature fluctuations in the composite Abrikosov lattice. Thermal fluctuations generate closed loops of composite vortices and *local* splitting of field-induced composite vortices. This phase features superfluidity, as well as longitudinal superconductivity. **Panel C:** A liquid of type-1 vortices immersed in a background lattice of type-2 vortices. There is a temperature region in low magnetic field when type-1 vortices form a vortex liquid and the corresponding vortex loops are proliferated, while type-2 vortices form an Abrikosov lattice. Type-1 and type-2 vortices carry only a fraction of magnetic flux quantum in this state. Superfluidity is lost and longitudinal superconductivity is retained. The arrow from panel B to panel C illustrates type-1 loop-proliferation. **Panel D:** A two-component vortex liquid of type-1 and type-2 vortices. Superfluidity and longitudinal superconductivity is lost, i.e. this is the normal metallic phase[5]. There is no type-2 loop proliferation going from panel C to panel D, since this is a first-order melting transition of the type-2 Abrikosov lattice *in the background of a liquid of linetension-less type-1 vortices.*

Consider now the processes illustrated in panel B of Fig. 1 for the case $|\psi^{(1)}|^2 \ll |\psi^{(2)}|^2$, upon increasing the temperature beyond the low-temperature regime. We may view this process as a type-1 closed vortex loop superposed on an Abrikosov lattice of (slightly) fluctuating composite vortices. An important point to notice is that a type-α vortex does not interact with a composite vortex by means of a neutral mode[6]. This follows from a topological argument that two split branches will feature nontrivial winding in the composite neutral field $\theta^{(1)} - \theta^{(2)}$, while a composite vortex line does not. Hence, the splitting transition may be viewed as *a type-1 vortex loop-proliferation in a neutral superfluid.* This is illustrated in Fig. 2. Thus, we may utilize the well-known results for the critical properties of the 3Dxy model for neutral superfluids described as a vortex-loop proliferation[12,13,14]. Therefore, somewhat counter-intuitively, this "vortex sublattice melting" phase transition is in the 3Dxy universality class[12,13,14], and not a first order melting transition. The resulting phase is one where superfluidity is lost and longitudinal superconductivity is retained in the component $\Psi_0^{(2)}$,[5] illustrated in panel C of Fig. 1.

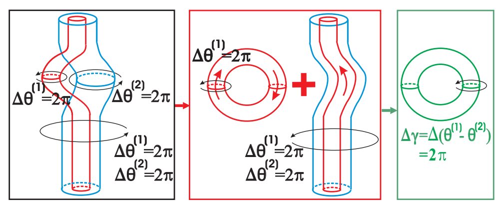

FIG. 2: Detailed illustration of the low-temperature thermal fluctuations in an Abrikosov lattice of composite vortices. A local excursion of the vortex component with lowest bare phase stiffness (type-1 vortex) away from the composite vortex lattice may be viewed as a type-1 bound vortex loop superposed on the composite Abrikosov lattice. The composite vortex line does not interact with a vortex with nontrivial winding in $\Delta\gamma = \Delta(\theta^{(1)} - \theta^{(2)})$[6]. A splitting transiton of the composite Abrikosov lattice, such as illustrated in going from panel B to panel C in Fig. 1, may therefore be viewed as a *zero-field* vortex-loop proliferation of type-1 vortices, with a phase transition in the 3Dxy universality class[12,13,14].

Apart from the sub-lattice melting transition, thermal fluctuations will produce a melting transition of the type-2 Abrikosov lattice at a higher temperature. It is well known that sufficiently strong thermal fluctuations drive *a first-order* melting transition of the Abrikosov lattice[14] in $N = 1$ superconductors. Due to the interplay with the proliferated type-1 vortices, a counterpart to this effect for the case $N = 2$ when $|\psi^{(1)}|^2 \neq |\psi^{(2)}|^2$ is more complex. The melting temperature $T_{\rm M}(B)$ of the type-2 Abrikosov lattice is suppressed with increasing magnetic field[14]. At low enough magnetic fields, upon heating the system, the 3Dxy type-1 vortex-loop transition at $T_{\rm c1}(B)$ will be encountered before the melting transition of type-2 vortices at $T_{\rm M}(B)$. Above $T_{\rm M}(B)$, longitudinal superconductivity is also lost, whence we may infer that the vortex-liquid mixture of liberated type-1 and type-2 vortices is the normal metallic phase[5], depicted in panel D of Fig. 1.

The above physical picture is borne out in large-scale MC simulations. We consider the model based on Eq. (1) for $N = 2$ on an $L \times L \times L$ lattice with periodic boundary conditions for coupling constants $|\psi^{(1)}|^2 = 0.2$, $|\psi^{(2)}|^2 = 2$, and $e^2 = 1/10$. The Metropolis algorithm with local updating is used in combination with Ferrenberg-Swendsen reweighting[15]. To capture the correct physics, system sizes as large as $L = 96$ are used. The external magnetic field $\mathbf{B}$ studied is $B^x = B^y = 0$, $B^z = 2\pi/32$, thus there are 32 plaquettes in the (x, y)-plane per flux-quantum. This is imposed by splitting the gauge field into a static part $\mathbf{A}_0$ and a fluctuating part $\mathbf{A}_{\rm fluct}$. The former is kept fixed to $(A_0^x, A_0^y(\mathbf{r}), A_0^z) = (0, 2\pi x f, 0)$ where f is the magnetic filling fraction, on top of which the latter field is free to fluctuate. Together with the periodic boundary conditions on $\mathbf{A}_{\rm fluct}$, the constraint $\oint_C (\mathbf{A}_0 + \mathbf{A}_{\rm fluct}) d\mathbf{l} = 2\pi f L^2$, where C is a contour enclosing the system in the (x,y)-plane, is ensured. Note that it is imperative to fluctuate $\mathbf{A}$, otherwise type-1 and type-2 vortices do not interact[3,4,6]. To investigate the criticality of the transition at $T_{\rm c1}$ we have performed finite size scaling (FSS) of the third moment of the action. These simulations are done by using vortices directly, similar to the procedure described in Ref. 4, the only difference being the finite magnetic induction of $B^z = 2\pi/32$.

We compute the specific heat and the third moment of the action $S = \int d\mathbf{r}\mathcal{L}$, defined as $M_3 = (\langle S^3\rangle - \langle S\rangle^3)/L^3$. The peak to peak value of M_3 scales with system size as $L^{(1+\alpha)/\nu}$ and the width between the peaks scales as $L^{-1/\nu}$[16]. To probe the structural order of the vortex system we compute the planar structure function $S^{(\alpha)}(\mathbf{k}_\perp)$ of the *local vorticity* $\mathbf{n}^{(\alpha)}(\mathbf{r}) = \left(\nabla \times \left[\nabla\theta^{(\alpha)} - e\,\mathbf{A}\right]\right)/2\pi$, given by

$$S^{(\alpha)}(\mathbf{k}_\perp) = \frac{1}{(fL^3)^2}\,\langle|\sum_{\mathbf{r}}\; n_z^{(\alpha)}(\mathbf{r})\; e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}|^2\rangle, \quad (3)$$

where $\mathbf{r}$ runs over dual lattice sites and $\mathbf{k}_\perp$ is perpendicular to $\mathbf{B}$. This function will exhibit sharp peaks for the characteristic Bragg vectors $\mathbf{K}$ of the type-α Abrikosov lattice and will feature a ring-structure in its corresponding liquid of type-α vortices. The signature of vortex sub-lattice melting will be a transition from a six-fold symmetric Bragg-peak structure to a ring structure in $S^{(1)}(\mathbf{K})$ while the peak structure remains intact in $S^{(2)}(\mathbf{K})$. Furthermore, we compute the *vortex co-centricity* $N_{\rm co}$ of type-1 and type-2 vortices, defined as $N_{\rm co} \equiv N_{\rm co}^+ - N_{\rm co}^-$, where

$$N_{\rm co}^{\pm} \equiv \frac{\sum_{\mathbf{r}} |n_z^{(2)}(\mathbf{r})|\delta_{n_z^{(1)}(\mathbf{r}),\pm n_z^{(2)}(\mathbf{r})}}{\sum_{\mathbf{r}} |n_z^{(2)}(\mathbf{r})|}, \quad (4)$$

where $\delta_{i,j}$ is the Kronecker-delta. The reason for considering this subtracted function is that we then eliminate the effect of random overlap of vortices in the high-temperature phase $T > T_{\rm c1}$ due to vortex-loop proliferation, and focus on the *compositeness* of field-induced vortices. Note that $\delta_{n_z^{(1)}(\mathbf{r}),\pm n_z^{(2)}(\mathbf{r})} = 1$ if and only if vortex segments of type-1 and type-2 are *locally* co-centered *and co-directed* $(+)$ and *counter-directed* $(-)$ on the same dual lattice site $\mathbf{r}$, otherwise it is zero. Hence, $N_{\rm co}$ is the fraction of type-2 vortex segments that are co-centered with type-1 vortices, providing a measure of the extent to which vortices of type-1 and type-2 form a *composite* vortex system. Hence, it probes the splitting processes visualized in panel B of Fig. 1 and in Fig. 2. The results are shown in Fig. 3.

The specific heat has a pronounced peak at $T_{\rm c1}$ associated with the 3Dxy transition, as well a broader and less pronounced peak which is the finite field remnant of the zero-field inverted 3Dxy transition[13]. Scaling of M_3 at $T_{\rm c1}$ shown in the inset c in Fig. 3 yields the critical exponents $\alpha = -0.02 \pm 0.06$ and $\nu = 0.67 \pm 0.05$ in agreement with the 3Dxy universality class. A novel result, not encountered in one-component superconductors, is that the structure function $S^{(1)}(\mathbf{K})$ vanishes *continuously* as the temperature approaches $T_{\rm c1}$ from below, which is precisely the hallmark of the decomposition transition that separates the two types of vortex states depicted in panels B and C in Fig. 1. A related intriguing feature is the *vanishing* in the co-centricity $N_{\rm co}$ at $T_{\rm c1}$ as a function of temperature, which we will discuss in detail below. The first-order melting transition is seen to take place at $T_{\rm M}$, where $S^{(2)}(\mathbf{K})$ is seen to vanish discontinuously. This is the temperature at which the translational invariance is restored through melting of the type-2 Abrikosov vortex lattice. The resulting translationally invariant high-temperature phase is depicted in panel D of Fig. 1. In the temperature interval $T < T_{\rm c1}$ the system features superconductivity and superfluidity simultaneously[5], since there is long-range order both in the charged and the neutral vortex modes. In the temperature interval $T_{\rm c1} < T < T_{\rm M}$ long-range order in the neutral mode is destroyed by loop-proliferation of type-1 vortices, hence superfluidity is lost[5]. However, longitudinal one-component superconductivity is retained, along the direction of the external magnetic field. For $T > T_{\rm M}$ superconductivity is also lost, hence this is the normal metallic state, which is a two-component vortex liquid.

Next we discuss these results in more detail. The most unusual and surprising feature is the continuous variation of $S^{(1)}(\mathbf{K})$ with temperature, even at $T_{\rm c1}$ where it

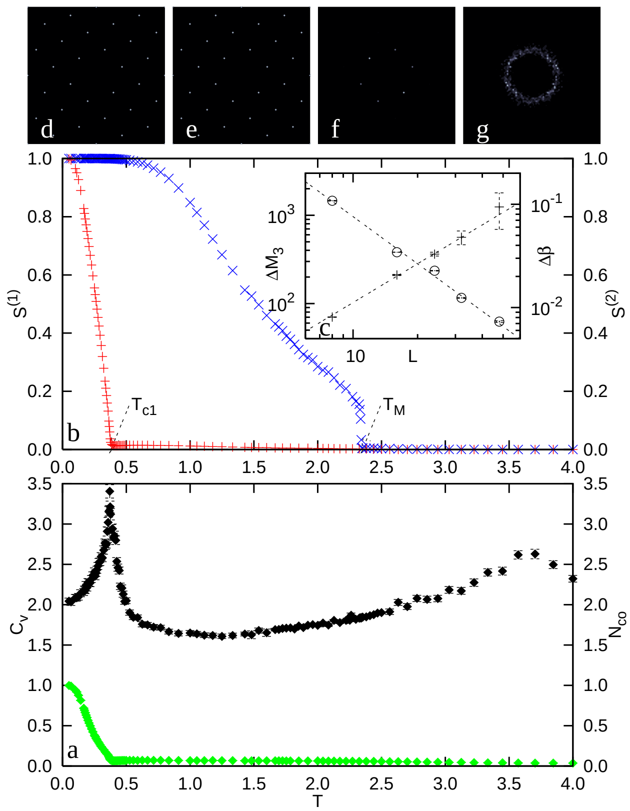

FIG. 3: MC results for $N = 2$ $|\psi^{(1)}|^2 = 0.2$, $|\psi^{(2)}|^2 = 2$, and $e = 1/\sqrt{10}$ are presented. Panel a shows specific heat C_v in black and co-centricity $N_{\rm co} = N_{\rm co}^{+} - N_{\rm co}^{-}$ in green. Note how the specific heat anomaly at $T_{c1} = 0.37$, associated with the proliferation of type-1 vortices, matches the point at which $N_{\rm co}$ drops to zero, illustrating that type-1 vortices tear themselves off type-2 vortices. The remnant of the zero-field anomaly in the specific heat can be seen as a hump at $T \sim 3.6$. Panel b shows the structure factors $S^{(1)}(\mathbf{K})$ in red and $S^{(2)}(\mathbf{K})$ in blue for the particular Bragg vector $\mathbf{K} = (\pi/4, -\pi/4)$. We see that $S^{(1)}(\mathbf{K})$ vanishes continuously at $T_{\rm c1}$, while $S^{(2)}(\mathbf{K})$ vanishes discontinuously at $T_{\rm M} = 2.34$. Panel c shows the FSS plots of the M3 from which the exponents $\alpha = -0.02 \pm 0.06$ and $\nu = 0.67 \pm 0.05$ is extracted, showing that the sub-lattice melting is a critical phenomenon in the 3Dxy universality class. Panels d, e, f, and g are gray shade plots of the structurefactor $S^{(2)}(\mathbf{k}_\perp)$ for the temperatures $T_{\rm d} = 0.35$, $T_{\rm e} = 0.4$, $T_{\rm f} = 1.66$, and $T_{\rm g} = 2.85$, respectively. At $T_{\rm d}$ and $T_{\rm e}$ on both sides of the specific heat anomaly, the Abrikosov FLL remains intact. The six fold Bragg symmetry is intact at $T_{\rm f}$, but is lost at $T_{\rm M}$ to give a ring pattern at $T_{\rm g}$, the hallmark of a vortex liquid.

vanishes. The explanation for this is the topological arguments introduced above, involving a vortex-loop proliferation of type-1 vortices (which destroys the neutral superfluid mode) in the background of a composite vortex lattice, which the type-1 vortices essentially do not see, cf. Fig. 2. As far as the composite neutral Bose field $\theta^{(1)} - \theta^{(2)}$ is concerned, *it is precisely as if the composite vortex lattice were not present at all.* Hence, $S^{(1)}(\mathbf{K})$ vanishes for a completely different reason than $S^{(2)}(\mathbf{K})$, namely due to *critical fluctuations, i.e. vortex-loop proliferation* in the condensate component with lowest bare stiffness. Such a phase transition does not completely restore broken translational invariance associated with an Abrikosov vortex lattice, since for the type-2 vortices *quite remarkably, the Abrikosov vortex lattice order survives the decomposition transition*, due to interaction between heavy vortices mediated by charged modes. The vanishing of $N_{\rm co}$ is particularly interesting, and finds a natural explanation within the framework of the above discussion. That is, for $T \ll T_{\rm c1}$, we have $N_{\rm co} \approx 1$, so the vortex system consists practically exclusively of composite vortices. As the temperature increases, thermal fluctuations induce excursions such as those illustrated in panel B of Fig. 1 and in Fig. 2, which reduce $N_{\rm co}^{\pm}$ from their low-temperature values, reaching a *minimum* at $T_{\rm c1}$ and then *increase* for $T > T_{\rm c1}$. For temperatures above, but close to, $T_{\rm c1}$ fluctuations in vortices originating in $\Delta\theta^{(2)}$ are still small, so the variations in $N_{\rm co}^{\pm}$ reflect thermal fluctuations in vortices originating in $\Delta\theta^{(1)}$. The increase of $N_{\rm co}^{\pm}$ means that type-1 vortex loops are thermally generated, and thus tend to *randomly* overlap more with the moderately fluctuating type-2 vortices. At their first order melting transition, type-2 vortices fluctuate only slightly. *Thus, the vanishing of* $N_{\rm co}$ *above* $T_{\rm c1}$ *reflects the increase in the density of thermally generated type*-1 *vortex loops in the background of a slightly fluctuating type*-2 *Abrikosov vortex lattice*, cf. panel C in Fig. 1.

In summary, we have investigated the orders and universality classes of thermally driven phase transitions in a two-component vortex system in a magnetic field. We find two phase transitions for the fields we consider in this paper (low-field regime). *i)* A 3Dxy phase-transition associated with melting of the Abrikosov lattice originating in the phases with lowest stiffness, which may be viewed as vortex-loop proliferation taking place in the background of a composite vortex lattice. The structure function associated with the type-1 vortices vanishes *continuously* at the transition. This phase transition has no counterpart in a one-component superconductor. *ii)* A first-order vortex lattice melting associated with restoration of translational invariance of the type-2 vortex system. The corresponding structure function vanishes *discontinuously* at the transition, which however takes place in the background of a *liquid of linetension-less type*-1 *vortices.* This also sets the type-2 Abrikosov lattice melting apart from the corresponding phenomenon in one-component superconductors.

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[1] J. Jaffe and N. W. Ashcroft, Phys. Rev. B **23**, 6176 (1981); *ibid* **B 27**, 5852 (1983); K. Moulopolos and N. W. Ashcroft, Phys. Rev. Lett., **66**, 2915 (1991); Phys. Rev. B **59** 12309 (1999); J. Oliva and N. W. Ashcroft, Phys. Rev. **B 30**, 1326 (1984); N. W. Ashcroft, J. Phys. **A129**, 12, (2000); Phys. Rev. Lett., **92**, 187002 (2004).
[2] E. Babaev, L. Faddeev, and A. Niemi, Phys. Rev. B., **65**, 100512 (2002).
[3] E. Babaev, Phys. Rev. Lett., **89**, 067001 (2002).
[4] J. Smiseth, E. Smørgrav, and A. Sudbø, Phys. Rev. Lett., **93**, 077002 (2004).
[5] E. Babaev, A. Sudbø, and N. W. Ashcroft, Nature **431** 666 (2004).
[6] J. Smiseth, E. Smørgrav, E. Babaev, and A. Sudbø, to be submitted to Phys. Rev. B.
[7] T. Senthil, L. Balents, S. Sachdev, A. Vishwanath, and M. P. A. Fisher, Science **303**, 1490 (2004); O. Motrunich and A. Vishwanath, Phys. Rev. B **70**, 075104 (2004); S. Sachdev, in *Lecture Notes in Physics*, U. Schollwock, J. Richter, D. J. J. Farnell, and R. A. Bishop eds. Springer, Berlin (2004).
[8] A. J. Niemi, K. Palo and S. Virtanen, Phys. Rev. D **61**, 085020 (2000).
[9] F. Datchi, P. Loubeyre, and R. le Toullec, Phys. Rev. B **61**, 6535 (2000).
[10] S. A. Bonev, E. Schwegler, T. Ogitsu, and G. Galli, Nature **431**, 669 (2004).
[11] T. Guillot, Physics Today **57**, 63 (2004).
[12] Z. Tesanovic, Phys. Rev. B **59**, 6449 (1999).
[13] A. K. Nguyen and A. Sudbø, Phys. Rev. B **60**, 15307 (1999).
[14] K. Fossheim and A. Sudbø, *Superconductivity: Physics and Applications*, John Wiley & Sons, London (2004).
[15] A. M. Ferrenberg and R. H. Swendsen, Phys. Rev. Lett., **61**, 2635 (1988); *ibid* **63**, 1195 (1989).
[16] A. Sudbø, E. Smørgrav, J. Smiseth, F.S. Nogueira, and J. Hove, Phys. Rev. Lett.,**89**, 226403 (2002); J. Smiseth, E. Smørgrav, F.S. Nogueira, J. Hove, and A. Sudbø, Phys. Rev. B, **67**, 205104 (2003).